Equational Hybrid Type Theory

María Manzano[1], Manuel A. Martins[2], Antonia Huertas[3]

[1] mara@usal.es-Dep. of Philosophy, University of Salamanca, Spain
[2] martins@ua.pt-CIDMA - Dep. Mathematics, Universidade de Aveiro, Portugal
[3] mhuertass@uoc.edu-Dep. of Comp. Science, Universitat Oberta de Catalunya, Spain

Abstract

The definition of identity in terms of other logical symbols is a recurrent issue in logic. In particular, in FOL there is no way of defining the global relation of identity, while in standard SOL this definition is not only possible, but widely used. In this paper the reverse question is posed and affirmatively answered: Can we define with only equality and abstraction the remaining logical symbols? Our present work is developed in the context of equational hybrid logic (i.e. a modal logic with equations as propositional atoms enlarged with the hybrid features: nominals and the @ operator). We take equality, λ abstraction, nominals and $\Diamond$ operator as primitive symbols and we will show that all of the remaining logical symbols can be defined, including propositional quantifiers and the @ operator.

Keywords: Equational hybrid logic, Type theory, Identity.

1 Introduction

The relation of identity is usually understood as the binary relation which holds between any object and itself and which fails to hold between any two distinct objects. Due to the central role this notion plays in logic, you can either be interested in how to define it using other logical concepts and which logic is required, or else in the opposite scheme; namely, in the definition of the other logical concepts (connectives and quantifiers) in terms of the identity relation, using also abstraction.

In first order logic (FOL) identity can not be defined in Equality-free FOL [4]. Even in the best scenario of a formal language with only a finite set of predicate constants, in which a binary relation obeying the usual rules for equality can be defined (i.e., expressing that two objects cannot be distinguished), we can find models where the interpretation of equality is not the identity.

In second order logic (SOL) identity can be defined by Leibniz's principle [4]. Thus, in Equality-free SOL there is a formula to define equality for individuals and the relation defined by this formula is 'genuine' identity in any standard second order structure. SOL with standard semantics has an extraordinary expressive power but poor logical properties and then, non-standard semantics

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