Explicatory Conceptions and the teaching of Logic.

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1. Introduction.

Karl R. Popper[1] points out the worth of *problems* in the teaching of philosophy and science. Popper's well-known epistemological thesis – *knowledge begins with problems* – proves to be useful according to this author in the pedagogical field. The question is naturally how these ideas should be implemented in the educational practice, and, in particular, in the teaching of logic. It will be suggested here that a certain form of understanding explicatory processes (which involve logical concepts) support – in some measure – an application of those ideas.

2. Explicatory Processes.

Certain intellectual efforts – in science and philosophy – are often considered attempts to "explicate" certain concepts taken as obscure or troublesome and, at the same time, worth of theoretical interest. For instance, the rigorous mathematical characterization of limit in terms of ä-å is an explication of the intuitive notion of the limit of a function. Let us call these analytic activities "explicatory processes". In very general, these can be characterized elementarily as following:

We will consider an *explicatory process* as an intellectual activity that aims to obtain out of an intuitive, vague, confusing notion a conceptual counterpart that is clearer and satisfactory (from a theoretical point of view).

This "intellectual activity" may be analyzed from a variety of viewpoints: historical, psychological, and methodological. If the interest is essentially the latter one, we can distinguish – following A. Coffa[2] -- two forms of conceiving this process: the *Quinean* conception and the *Tarskian*[3] conception. Explications – considered from the first perspective – would consist, basically, of constructing certain expressions – explicata – capable of substituting or replacing troublesome but useful expressions – explicanda – in order to preserve their use in certain contexts (those in which they are useful) and to achieve – arbitrarily – precision in the rest of the contexts. The central notions here are use and usefulness – without appealing to the meanings involved. Quine expresses this with his proverbial clearness[4]:

> Explication is elimination. We have, to begin with, an expression or form of expression that is somehow troublesome. It behaves partly like a term but not enough so, or it is vague in ways that bother us, or it puts kinks in a theory or encourages one or another confusion. But also it serves certain purposes that are not to be abandoned. Then we find a way of accomplishing those same purposes through other channels, using other and less troublesome forms of expression. The old perplexities are resolved.

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[1] See Popper, K. [1974]
[2] See Coffa [1975]
[3] Perhaps it would be fairer to call it "Tarski-Coffa"
[4] Quoted by Coffa (ob. Cit.) page 46. Word and Object page 260

The explications – from the second point of view referred – would pretend to establish certain relations between meanings or concepts: a vague or ambiguous concept – associated to the explicandum – and a clear and precise concept – associated to the explicatum. To explicate, then, is to construct a rigorous concept that pretends to capture what the previous intuitive concept expressed vaguely. The superiority of this latter conception dwells – according to Coffa – in the fact that it accounts for the ability of explications to "find solutions" to problems and not merely to "shift" those problems. This asset would emerge from the vigorous connection existing – in the Tarskian perspective – between what is referred by the explicandum and the explicatum.

The preceding description (surely insufficient) perhaps enables to envisage the contrast between both conceptions; nevertheless, our present interest is not that comparative analysis. In the following pages, thus, we will center our attention exclusively in the Tarskian conception. First, it is described thoroughly and then we suggest how it can be exploited in didactic terms.

3. The Tarskian Model

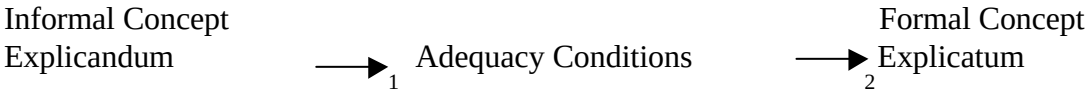
The central point of this model is – as it was said – the assumption that explicating consists of establishing certain relations between meanings or concepts. The following passage of Tarski is eloquent[5]:

> The problem set in this article belongs in principle to the type of problems that frequently occur in the course of mathematical investigations. Our interest is directed towards a term which we can give an account that is more or less precise in its intuitive content, but the significance of which has not at present been rigorously established, at least in mathematics. We then seek to construct a definition of this term which, while satisfying the requirements of methodological rigor, will also render adequately and precisely then actual meaning of the term.

A central aspect of this model consists of holding a very strong connection between the respective concepts and, consequently, the implicit proposal of a evaluation criterion of the correction of the explicatory operation: the new concept must help to clarify, precisely, those misty intuitions which went along with the old concept.

Coffa outlines such process by differentiating three stages. First, the concept of which we have an intuitive and vague notion (explicandum) is identified. Second, - by a partial analysis of the explicandum - certain conditions that any concept which is put forward as an explication must satisfy, are established; and lastly, a rigorous concept (explicatum) that satisfies the previous conditions is proposed.

Applied to the specific field of mathematics, perhaps the former ideas can be represented graphically in the following way:



The first stage would possess a definitional status and, therefore, it would not be susceptible of criticism since it would simply consist of the identification of the meaning of an expression that will be object of analysis – remember its vague character

[5] Quoted by Coffa (ob.cit.) The passage is in Tarski, "On definible sets of real numbers" (1930) in Logic, Semantics, Metamathematics, Clarendon Press, 1956, pp. 111-112.

justifies the explicatory effort. Besides, ¿what would we compare or contrast the concept with? This stage is, precisely, the beginning of the analysis.[6]

The second stage, conversely, would enable the evaluation, in so far as those adequacy conditions are determined by the partial analysis of the explicandum. Perhaps it can be said that it deals with features or properties of the concept object of explication and therefore those assertions should be assessed in terms of truth and falsehood.

It is important to notice that there are two "types" of relations – sub-1 and sub-2 – that concern both the *description* and the *evaluation* of the explicatory processes. The first "type" is related to questions such as the possibility of capturing – by means of the establishment of certain more or less general conditions – those properties that any concept proposed as explicatum of the explicandum in question, should necessarily satisfy. The second "type" bears problems such as the evaluation of the explicatum in terms of the fulfillment of the adequacy conditions.

4. Tarskian Model and the Teaching of Logic.

This model calls the attention on two aspects especially interesting – from the point of view of the *motivation* – in the teaching of logic. These aspects are:

a) The *criticism* of the formal concepts (by reason of the analysis of the conceptual relations of both "types")
b) The *justification* of formal concepts (by reason of the analysis of the conceptual relations of both "types")

The *motivational* interest mentioned above results from the fact that an approach of this nature enables the integration, in a significantly articulated fashion, of the *mathematical* scope, the *philosophical-conceptual* and the *historical* scope. Not every logic topic is susceptible of this kind of treatment, but it must be admitted that many key concepts are perfectly fitted for it. The following is a quick description of a paradigmatic case.

The *criticism* that Tarski develops in his famous writings on logical consequence[7] to the notion of syntactic consequence can be understood as an evaluation of the relations between the pre-formal concept of consequence and the formal one. To say it in a few words, Tarski spots some sort of "extensional failure" in the concept of syntactic consequence against the pre-formal concept. Such a criticism is *philosophically* interesting and at the same time *historically* valuable and it can work as a *motivation* for the construction of the alternative model-theoretical (concept of semantical consequence)

This remarkable construction must, in turn, be *justified* and such an effort is based in the relations between this and the pre-formal concept (of consequence). The relations between both notions is – distinctly – an unsettling question of the *philosophy of logic* as the due attention to these notions gives rise to stimulating *problems of a historical*

[6] The justification of explicating a concept is outside the explicatory process itself. However, that justification can obviously be subject of critics: it only makes sense to explicate concepts that are not only obscure but also philosophically or scientifically relevant.
[7] See Tarski [1936]

nature.[8] From the philosophical point of view, those inquiries rebound dramatically in the meaning that should be given to logical properties as completeness, Löwenheim-Skolem and compacity (and their "counterparts" in superior orders). From the historical point of view, these pursuits can help to understand, for instance, the proclivity for first order.

This is an especially fascinating case, but it is not the only one. The intuitive notion of "computability" and its different formal counterparts also is an auspicious field for this motivating intersection of scopes. Gentzen's natural deduction calculi are another stimulating endeavor susceptible of a treatment in explicatory terms, as well. In short, the explicatory model referred, as long as it is capable of stimulating conceptual attention towards the relations between pre-formal concepts and the adequacy conditions (more or less explicitly formulated) and formal concepts invites to combine – in the appropriate pedagogical doses – mathematical, conceptual and historical elements in the teaching of logic; turning the technical developments more attractive to those less sensible to the mathematical beauty, and generally enlightening the comprehension of the context in which those developments were created.

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[8] An analysis in explicatory terms of Tarski [1936] can be seen in Seoane [1998]. The philosophical discussion and the significant historical problems referred can be seen (among others) in Kreisel [1965] and Shaphiro [1991].