Introducing reflections on relevance logic in introductory courses.

It is generally admitted that the truth-functional interpretation of
implication does not account for many ordinary expressions of conditional
thought. But neither should we be oblivious of the fact of its failure of
adequacy for the analysis of scientific explanation. As material
implication is validated by the independent truth of the consequent or the
falsity of the antecedent, explanation (in which a final condition is
deduced from the conjunction of a theory and the statement of initial
conditions) is trivialized simply by the truth of the statement on the
final condition if we adjust the notion of deduction in such a way that it
should be equivalent to material implication. If an explanation is not
trivial, there is in use a notion of deduction in which the facts mentioned
in the antecedent must be relevant for deducing the consequent.
The very arguments by which W.E.Johnson has shown that the "paradoxes of
implication" are harmless, show also the inadecuacy of the truth-fuctional
interpretation for deductive purposes: if the assertion of the truth of
implication is based on the falsity of the antecedent, then it cannot be
used in modus ponens; if it is based on the truth of the consequent, it
cannot be used in modus tollens and its use in modus ponens is redundant.
Therefore, concluded Johnson, material implication can be used in deduction
only if its assertion is not based on the independent truth values of the
antecedent and the consequent.
This condition was defended by G.H.von Wright as the "intensional"
assertibility of material implication, after showing the inadequacy of
C.I.Lewis' strict implication, which is based on the independent modalities
of the connected propositions. By contrast, von Wright's theory of relative
modalities is structurally akin to his conception of an intensional
valuation of material implication, but he is carefull in stating that each
of these theories is self-contained, so that the cross-reference is not
necessary.
In von Wright's way it is thus possible to give an account of relevant
implication (=entailment) without introducing new connectives. We can say
that the logic which constitutes the object language remains unchanged,
while the study of relevance is done in a metalenguage.
However, we must take notice of the fact that even in the current
presentations of classical logic the underlying ideas of deduction are not
unitary. In natural deduction implication is introduced by the relation of
deduction, and only by an ad-hoc adjustment (admitting within a secondary
proof the reiteration of an initial premiss alien to it, a procedure which
is equivalent to a technique of dilution+exportation) is this notion made
to be equivalent to the definition of implication in truth-functional
terms.
For this reason it may be convenient to include in introductory courses
variants
of natural deduction which open the spectrum of possibilities and show the
different uses of the correspondent notions of deduction. This may be done
in connection whith the determination of the premisses which are necessary
in order to reach a conclusion, which is essential in mathematics as well
as
in the revision of ordinary arguments. In fact, it must be kept in mind
that not only the notion of explanation in the empirical sciences does
require a relevant concept of deduction , but that also in mathematics the
distinction between premisses which are indispensible within a certain
proof and those which come out to be ocious is a strictly logical one,
which cannot be adjudicated to merely conversational implicatures.

The concept of relevant deduction is equally incident in the treatment of
disjunction. Anderson and Belnap maintained that if disjunction is
undersood extensionally, as when it is introduced by means of a-->avb,
then the disjunctive syllogism is not valid. If, on the contrary,
disjunction is understood intensionally, so that the negation of one of

the two terms implies the other one, then addition is not valid.
Their arguments are controversial and must not be accepted without
qualifications. But it is by all means important in the teaching of logic
to allow for the distinction of the different logical ideas involved in a
codification: Addition (which has a clear meaning in an extensional
context) (1); membership to an equivalent class to some effect, as it
is exemplified by the rule of disjunction-introduction in the antecedent
(2); inferring from X that ¬a-->b (3), and the division of a genus in
mutually exclusive species, which is equivalent to the mathematical notion
of partition (4), are all different ideas whose relations are open to
investigation. As in the case of if__then...., it is important to make
clear the different contexts which motivate their different meanings and to
find out their corresponding logical behavior.
Under this perspective relevance logic is not a different logic, but a
different way of conceptualizing the commonly admitted logical structures,
as the logical ideas involved in the truth-functional definition
of the connectives differ from their presentation in the inferential
approach of natural deduction by means of rules which specify the
conditions under which their introduction is an authorized consequence and
may in turn be exploited deductively.